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Understanding The Pythagorean Theorem Through The Area of a Square

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Abstrak. Pyhtagoras' Theorem is a very important principle in mathematics, but many elementary students find it difficult to understand it. Therefore, this study aims to describe the quality of student responses in finding the Pythagorean formula. It is a descriptive-qualitative study to obtain a collection of students' thought processes and actions. The research subjects were selected based on their abstraction characteristics which could be classified into five groups. They are pre-structural, unistructural, multistructural, relational and extended-abstract. Students who become research subjects are elementary school students in Bengkulu City, Indonesia. The subjects of this study were twenty five people. The research instrument is the researcher himself who is guided by student assignments. Task-based interviews were applied to collect complete data. It is data collection using audio-visual recording. This data collection technique ensures that the data collected is complete and accurate. It is the genetic decomposition of the research subject. The results of this study are 3 students with pre-structural character, 5 unistructural students, 9 multistructural students, 7 relational students and 1 extended-abstract student. The conclusion of the study is that there are relational students who are able to coherently combine separate pieces of information and find the Pythagorean formula.

Key words: Pythagorean theorem, understanding, response quality

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INTRODUCTION

Mathematics is a subject that is not easy for elementary students. It requires proper management. Based on the writer's experience, one of the principles they had difficulty learning was the Pythagorean formula. Pyhtagoras' Theorem is a very important principle in mathematics, but many elementary students find it difficult to understand it. This principle is contained in a right triangle. The concept and principle of a right triangle has developed since the days of Ancient Egypt until now (Wijayanti & Hidayanti, 2019). The Pythagorean Theorem has caught the attention of mathematicians, as well as its use (Putra, 2020). It is very useful for solving problems in everyday life.

The concept of a triangle is understood by preschoolers as a visual triangle. It's based on real things. Also, in the form of a three-sided figure, namely a drawing scheme with certain observable attributes (Sarama & Clements, 2016). This scheme is stored in the child's memory, and carried over to the elementary school level. They understand the triangle as a shape, others understand it as a combination of three sides. Regarding the right triangle and the Pythagorean Theorem, students can see from these two points of view. Research result (Huang & Leung, 2002), The student understands that the total area of a square constructed from adjacent sides (right triangles) is equal to the area of a square constructed from the hypotenuse. Learning through media can improve understanding of the Pythagorean theorem. It is learning that can increase students' awareness and interest in learning (Putra, 2020). Also, problem solving skills require a good critical thinking process (Prayitno, 2018).

According to (Atteh, Acquandoh, Boadi, & Andam, 2020), students are able to identify shapes that describe the Pythagorean theorem. It is a square construction on each side of a right triangle. Students using direct material in the intervention process can help improve student understanding and performance in learning about the Pythagorean theorem. Learning by utilizing local culture and direct objects can improve students' mathematical abilities (Widada, Herawaty, Beka, Sari, & Riyani, 2020)(Widada, Herawaty, Rahman, Yustika, & Elsa, 2020). Learning is interactive when it is well designed. Students are guided to carry out the abstraction process in learning mathematics (Nurhasanah, Kusumah, & Sabandar, 2017).

Students make use of right triangle units through new situations. They reflect the conception of the memorization area measurement. It is challenging students to adapt and expand the area square formula to cope with new situations (Kara, Eames, Miller, Cullen, & Barrett, 2011). Students present through context-based learning. It can be through real objects as well as the concept of a triangle.

According to different traditional learning contexts in the abstraction process of students. They reconstructed the triangle concept demonstrated by the teacher. Students are able to identify and solve problems about triangles (Nurhasanah et al., 2017). It is an abstraction process that students can do in understanding triangles. The quality of the abstraction process can be classified in Taxonomy. That is the SOLO Taxonomy (Structure of Observed Learning Outcome) (Biggs, J.B., and Collis, 1982). SOLO taxonomy is designed as an evaluation tool for the quality of student responses to assignments (Biggs & Collis, 2004). It includes five levels, namely prestructural, unistructural, multistructural, relational and extended abstract (Biggs, J.B., and Collis, 1982). They describe that structural students cannot do the assigned task or carry out tasks using irrelevant data. Unistrutural students can use a single piece of information in response to a draw. Multistructural students are able to make use of several pieces of information but cannot link them together. Relational students can combine separate pieces of information for the completion of a task. The extended abstract student can generate general principles from unified data that can be applied to new situations. Based on this classification, we are interested in discussing the abstraction ability of students in finding the Pythagorean formula.

METHODS

We wanted to know in depth about the students' abstraction ability in finding the Pythagorean formula. It is a student's thought and action process that can be described by the quality of the response in the paper-and-pencil activities and in-depth interviews. Therefore, we conducted a qualitative-descriptive study to obtain a collection of students' thought processes and actions. The research subjects were selected based on their abstraction characteristics which could be classified into five groups. They are pre-structural, uni-structural, multi-structural, relational, and extended-abstract. Students who become research subjects are elementary school students in Bengkulu City, Indonesia. The subjects of this study were twenty five people. The research instrument is the researcher himself who is guided by student assignments. Task-based interviews were applied to collect complete data. It is data collection using audio-visual recording. This data collection technique ensures that the data collected is complete and accurate. Transcription, reduction, verification and drawing conclusions are carried out in the data collection process. It is done through the genetic decomposition of the research subject. With this flow, we can conclude the thinking characteristics of students in finding the Pythagorean formula.

RESULTS AND DISCUSSION

Students understand the Pythagorean theorem in various ways. The research data shows that there are students who use a square area as one of them. We give the research subjects the flexibility to determine which method they use. Based on paper-and-pencil data and interviews with 25 study subjects, they can be classified into five levels of response quality. It can be seen in Figure 1.

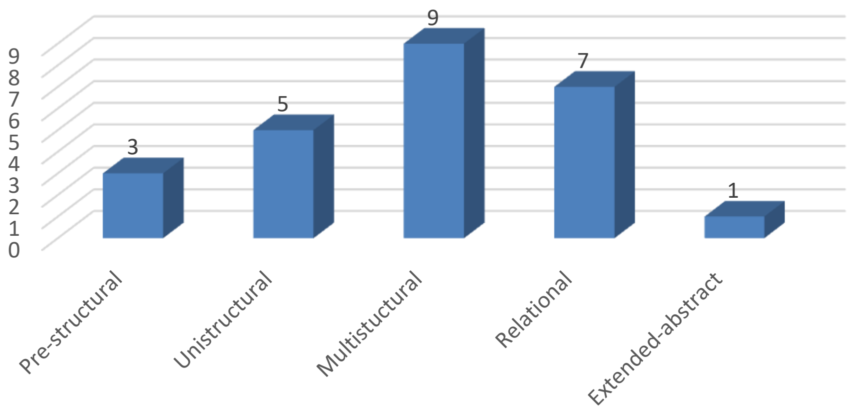

Figure 1. Frequency of students' understanding of the Pythagorean Theorem

Based on Figure 1, the quality of student responses in understanding the Pythagorean theorem. The data shows that there are 3 students with pre-structural character, 5 unistructural students, 9 multistructural students, 7 relational students and 1 extended-abstract student. Based on the genetic decomposition of each group, the description of each level is as follows. Prestructural students do not use data in trying to find the Pythagorean formula. Unistructural students can use a single piece of information (a single piece of data) in trying to find the Pythagorean formula. Multistructural students can use some pieces of information but it is not coherent. Relational students are able to coherently combine separate pieces of information and find the Pythagorean formula. The extended abstract student can generate general principles from unified data and can demonstrate in other ways. Next, we will show students relational cognitive activity. There were 7 relational students in this study, we selected one of them. The student is Yr. He represents his assignment in paper-and-pencil as shown in Figure 2.

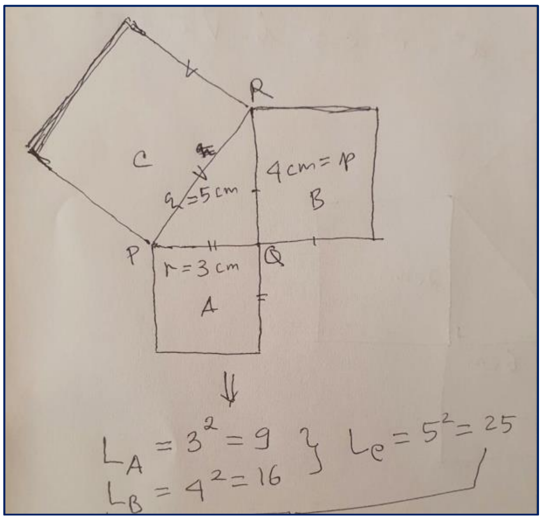

Figure 2. Yr represents the property of a special right triangle

Figure 2 shows that Yr tries to represent three squares A, B, and C, with side lengths of 3 cm, 4 cm, and 5 cm, respectively. It is the side length of the triangle PQR right. PQ = r = 3 cm, QR = p = 4 cm, and PR = q = 5 cm. The area of each square A is 9, square B is 16 and C is 25 with units of square cm.This is supported by data from interviews with Yr as follows.

Q: Notice that the PQR triangle is the right angle in Q. Can you explain the results of your paper-and-pencil?

Yr: Yes Sir, suppose the length of PQ = r, QR = p and PR = q, then r = 3 cm, p = 4 cm, and q = 5 cm.

Q: Ok… continue…

Yr: Based on some references and I saw the broadcast on the YouTube channel, I made 3 rectangles, each side length being r, p and q. It is square A with sides of 3 cm, square B with sides of 4 cm and square C with sides of 5 cm.

Q: Alright ... how about the next explanation ...

Yr: I got the area of each square,… The area of each square A is 9, square B is 16 and square C is 25 in cm square.

Q: …… hm

Yr:… I found the relationship is Area C equal to Area A plus Area B.

Based on the interview excerpt and Figure 2, Yr shows the quality of the response to his assignment is quite good. He was able to combine information about the sides of a right segment by taking advantage of the area of the square, each side of which is the side of the triangle. The side that is 3 cm long, the side 4 cm, and measuring the other side, which turns out to be 5 cm long. Yr make three squares on each side. He found that the area of the square on the hypotenuse is equal to the sum of the other two areas of the square. Yr concluded that 32 + 42 = 52. Furthermore, consider Figure 3.

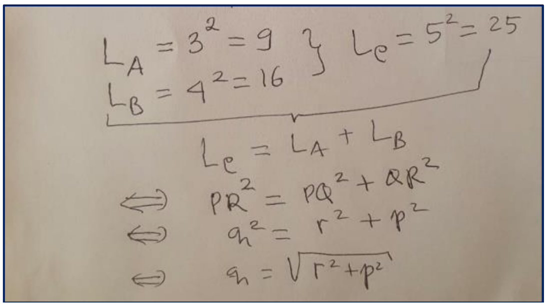

Figure 3. Continued process of discovering the Pythagorean formula by Yr

Yr was able to make Pythagorean formulas with precision. He makes use of the area A (= LA), the area B (= LB) and the area C (= LC). That is the property it gets, namely LC = LA + LB. We convey excerpts of our interview with Yr as follows.

Q: Try to explain the further!

Yr: Ok Sir, ... I got the area C = the area A + the area B ... so LC = LA + LB

Q: Ok OK ...

Yr: We look again at the right triangle PQR which is right at Q, then the area C = PR^2, the area A = PQ^2, and the area B = QR^2.

Q:….

Yr:… therefore, $PR^2 = PQ^2 + QR^2$,… and PR = q, PQ = r, QR = p. Thus $q^2 = r^2 + p^2$…

Based on the interview excerpt and Figure 3, Yr was able to combine information about the sides of the triangle and the area of the square whose side is the side of the triangle. This combination forms a property that in a right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the other sides. He was able to produce a principle about the relationship between the sides of a right triangle in the case for the length of the sides 3, 4 and 5 units of length (cm). However, it was not only under special conditions that Yr was able to develop the properties of the triangle to become more general. Look at Figure 4, and a snapshot of the interview with the student.

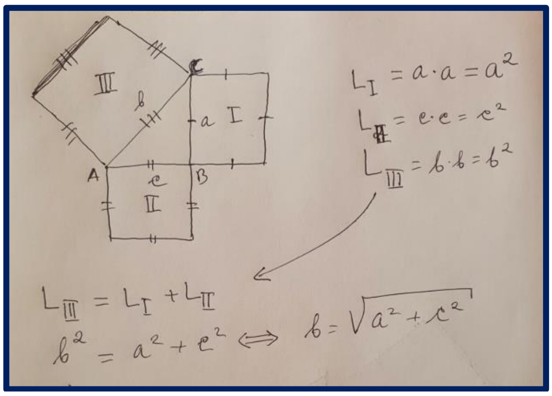

Figure 4. Yr found the Pythagorean formula

Figure 4 shows that Yr was able to achieve the general Pythagorean formula. It is a process of abstraction at the relational level. Based on Yr's process of discovery of the Pythagorean Theorem provides a good description of cognitive processes. He is understanding coherently. Actions, processes and objects regarding the area of a square and their relationship build a mature schematic of the Pythagorean Theorem. This can be seen from the YouTube channel with the following link: https://www.youtube.com/watch?v=xS64LNobG0s&t=5s. Based on the description of the channel, the sequence of steps in Figure 5 to Figure 9 can be presented.

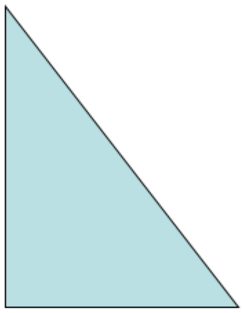

Figure 5. Right triangle

Figure 5 is a right triangle, through which three squares are made of the same side length as the side of the triangle. That is the new shape that can be seen in Figure 6.

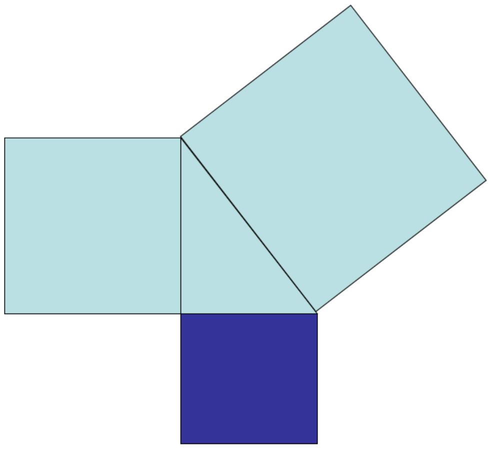

Figure 6. Three squares whose sides match the sides of the triangle

Three squares from each side of the triangle can be arranged into new sections. That is the image that can be seen in Figure 7. The figure shows that the rectangle of the red and blue rectangles can be affixed to the largest square.

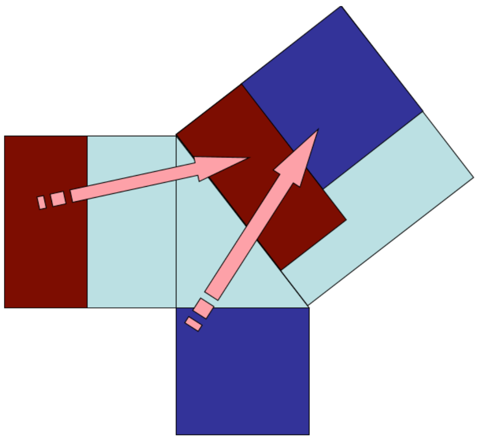

Figure 7. Glue the sections of the small

rectangle on the largest square

Figure 7 then proceeds to arrange the small parts of the building and attach them to the large square. It can be done and obtained as in Figure 8.

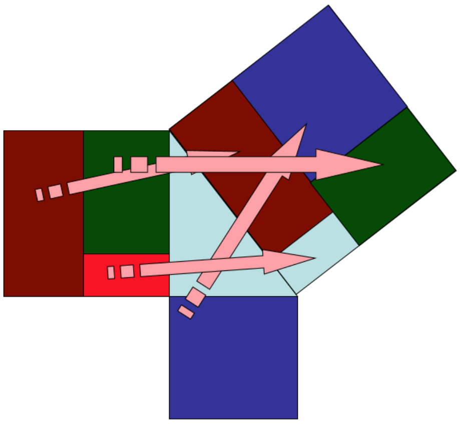

Figure 8. The area of a large square is equal to two small squares

Figure 8 represents the area of the small rectangles that cover the large rectangle. The area of the first small square is a^2 and the second small square is b^2. The two squares attach to the large square. The large square has an area of c^2. It gets the relationship that the area of a large square is equal to the sum of two areas of a small square. Therefore $c^2 = a^2 + b^2$, see Figure 9. That is the Pythagorean formula.

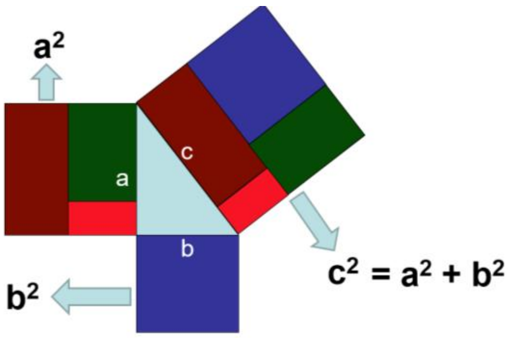

Figure 9. The area of a large rectangle is the same as the sum of the other two squares

Based on Figure 9, students achieved a correct abstraction process. Students are able to combine separate pieces of information for the completion of an assignment. That is information about the side of a right triangle. Also, the area of a square whose sides are equal to the sides of the right triangle. Students are able to connect three square areas correctly, that is, the area of a large square is equal to the sum of two areas of a small square. It is $c^2 = a^2 + b^2$. Students holistically reach a principle about the Pythagorean theorem.

The students have achieved a holistic abstraction process. Other research results support this research, among others, that students who are able to integrate pieces of information in an integrated manner are relational. Additional, and sometimes unnecessary, structures were found that existed before the results were repaired (Chick, 1998). Relational students provide several solutions, provide an explanation of the relationship between possible solutions, and explain the state of each solution to choose the best (Newton & Martin, 2014)(Jurdak, 1991)(Chan, Tsui, Chan, & Hong, 2002). Relational students make responses of more than one element that are coherently integrated for certain cases, for abstract levels of responses for more than one element which are coherently integrated and make analogies for other cases, but have not yet formed a new structure. (Potter & Kustra, 2012)(Putri, Mardiyana, & Saputro, 2017). Thus, we find students who are at the relational level. They are able to combine separate pieces of information in a coherent manner. It was the process of discovering the Pythagorean formula.

CONCLUSION

Representation and abstraction processes carried out by students can be classified through SOLO Taxonomy. This study found relational students who discovered the Pythagorean formula by combining pieces of information coherently. It is obtained through the process of abstraction from several real contexts. Therefore, we suggest using real contexts to represent and abstract the context into mathematical concepts or principles.

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